МІНІСТЕРСТВО ОСВІТИ І НАУКИ УКРАЇНИ
ДЕРЖАВНИЙ ВИЩИЙ НАВЧАЛЬНИЙ ЗАКЛАД
«УКРАЇНСЬКИЙ ДЕРЖАВНИЙ ХІМІКО-ТЕХНОЛОГІЧНИЙ
УНІВЕРСИТЕТ»

МЕТОДИЧНІ ВКАЗІВКИ ДО ПРАКТИЧНИХ ЗАНЯТЬ З ДИСЦИПЛІНИ "ВИЩА МАТЕМАТИКА" ЗА РОЗДІЛОМ "АНАЛІТИЧНА ГЕОМЕТРІЯ НА ПЛОЩИНІ" ЗА ОСВІТНІМ РІВНЕМ "БАКАЛАВР" ДЛЯ ІНОЗЕМНИХ СТУДЕНТІВ УСІХ ФАКУЛЬТЕТІВ (АНГЛІЙСЬКОЮ МОВОЮ)

METHODICAL INSTRUCTIONS FOR PRACTICAL CLASSES IN THE DISCIPLINE "HIGHER MATHEMATICS" UNDER THE SECTION "ANALYTICAL GEOMETRY ON A PLANE" BY EDUCATIONAL LEVEL "BACHELOR" FOR FOREIGN STUDENTS OF ALL FACULTIES.

Затверджено на засіданні
кафедри вищої математики
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Методичні вказівки до практичних занять з дисципліни "Вища математика" за розділом "Аналітична геометрія на площині" за освітнім рівнем "Бакалавр" для іноземних студентів усіх факультетів (англійською мовою)

Methodical instructions for practical classes in the discipline "Higher Mathematics" under the section "Analytical geometry on a plane" by educational level "Bachelor" for foreign students of all faculties. / Укл.: Олевський В.І., Олевська Ю.Б.,Науменко Т.С., Шапка І.В. – Д.: ДВНЗ УДХТУ, 2019. – 18 с.

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Навчальне видання

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KEY ISSUES OF THE DISCIPLINE PROGRAM ON THE THEME "ANALYTICAL GEOMETRY ON A PLANE"

1. Coordinate systems on a plane

Coordinates converting using parallel transfer. Polar coordinate system. Equations of some curves in a polar coordinate system.

2. Ways of a straight line setting on a plane

General equation of a line and its research. The canonical and parametric equation of a straight line. Equation of a straight line passing through two points. Equation in segments. Equation of a straight line with angular coefficient. Parallelism and perpendicularity conditions of two straight lines. The angle between two straight lines. Equation of a bundle of straight lines. Normal equation of a straight line. Distance from point to line.

3. Second-order curves, their equations and properties

General equation of the second order line. Circle. Ellipse. Hyperbola. Parabola. Properties of lines of the second order and investigation of their form.

APPROXIMATED QUESTIONNAIRE FOR THE TOTAL KNOWLEDGE CONTROL

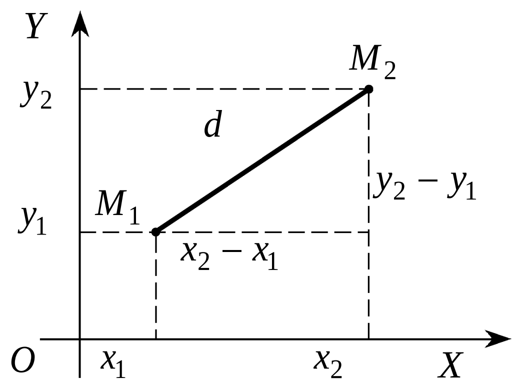
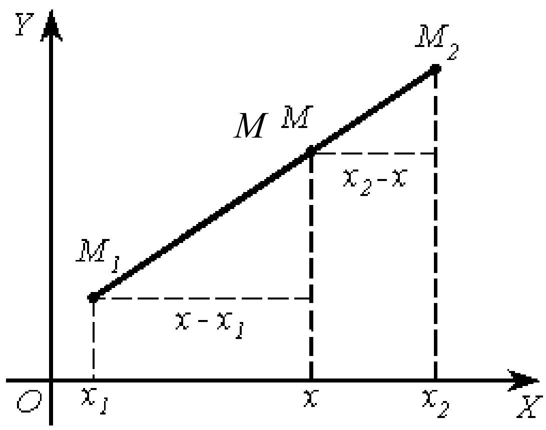
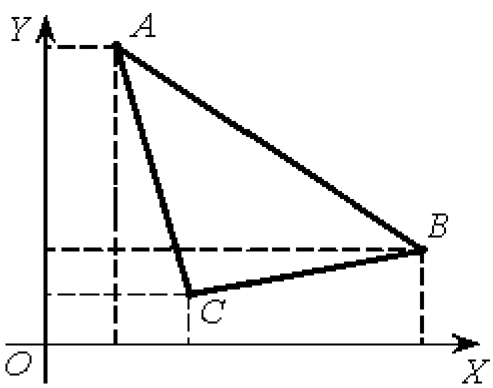
1. Rectangular Descartes' and polar coordinate systems. The connection between them.
2. Transformation of coordinates.
3. The distance between two points on a plane. Separation of a segment in a given relation.
4. Ways of setting a straight line on a plane.
5. General equation of a straight line, its research. The straight line through two points. Equation of straight line in the segments.
6. Linear equation with angular coefficient. A bundle of straight lines. Normal equation of a straight line.
7. The angle between two straight lines on the plane, the conditions of parallelism and perpendicularity for two straight lines.
8. Distance from the point to the straight line.
9. Polar parameters of the straight line. Normal equation of a straight line.
10. Circle. Equation of a circle.
11. Ellipse. Canonical equation of ellipse.
12. Canonical equation of hyperbole. Asymptotes of hyperbole. Equilateral hyperbole.
13. Canonical parabola equation.
14. Eccentricity and focal radii of ellipse and hyperbola. The general property of second-order curves.

1. ANALYTICAL GEOMETRY ON A PLANE

1.1. Cartesian rectangular coordinate system on a plane

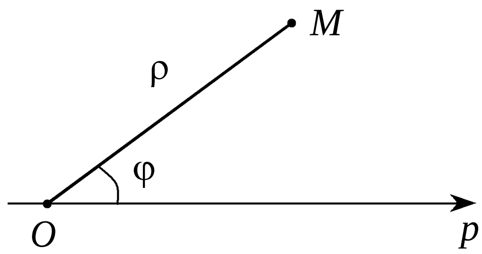
Numbers taken in a certain order, which determine the position of a point on a straight line, or on a plane, or in space or on a surface, are called **the coordinates of a point.** *Cartesian coordinates* are most often used.

Cartesian rectangular coordinate system on a plane – these are two mutually perpendicular coordinate axes Ox and Oy with the selected scale unit. Position of the point in the system xOy is given by a pair of numbers x, y.

	The distance between two points $M_1(x_1, y_1)$ i $M_2(x_2, y_2)$: $$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
	Separation of the segment M_1M_2 in the given relation λ is $$x=\frac{x_1+\lambda x_2}{1+\lambda};\ y=\frac{y_1+\lambda y_2}{1+\lambda}.$$ Coordinates of the middle of the segment M_1M_2 (where $\lambda=1$) are $$x=\frac{x_1+x_2}{2};\ y=\frac{y_1+y_2}{2}$$
	Triangle area ABC, where $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ – the coordinates of its vertices is $$S=\pm\frac{1}{2}\cdot\begin{vmatrix} x_1-x_3 & y_1-y_3 \\ x_2-x_3 & y_2-y_3 \end{vmatrix}$$

1.2. Polar coordinate system

The most important except rectangular coordinate system is **the polar coordinate system**. It is given by a point O, which called **a pole**, and a beam Op beginning at the pole and called **the polar axis**. Also units of scale have to be set : *linear* – to measure lengths of sections and *angle* one – to measure angles.

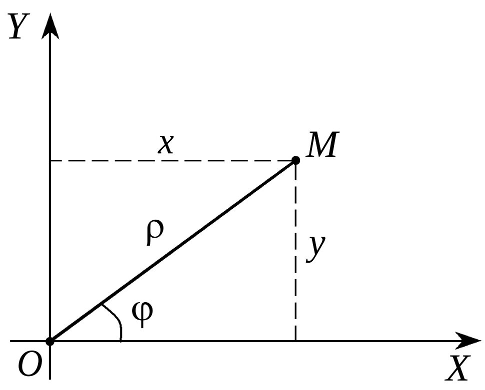


Consider a polar coordinate system and take an arbitrary point on a plane M. Suppose that $\rho = \left|\overrightarrow{OM}\right|$ – distance from point O to point M and $\varphi = \angle\left(Op, \overrightarrow{OM}\right)$ – the angle at which you have to turn the polar axis counterclockwise to coincide it with the vector $\overrightarrow{OM}$.

The numbers ρ and φ are called the **polar coordinates** of the point M. In this case, the number ρ is considered the first coordinate and is called the ***polar radius***, and the number φ is the second coordinate and is called ***the polar angle***. Point with polar coordinates is denoted as follows: $M(\rho;\varphi)$. The polar radius may acquire arbitrary nonnegative values $0 \le \rho < \infty$, obviously, and the polar angle is considered to be changing within $0 \le \varphi < \infty$.

Relationship between rectangular Cartesian and polar coordinates

Let's express the Cartesian coordinates of the point M through the polar one. We assume that the beginning of the rectangular system coincides with the pole, and the axis Ox – with the polar axis Op. If the point M has Cartesian coordinates x and y ($M(x,y)$) and polar ρ and φ ($M(\rho;\varphi)$), then



formulas for the transition *from polar coordinates to Cartesian ones*:

$$x = \rho\cos\varphi,$$
$$y = \rho\sin\varphi,$$

formulas for the transition *from Cartesian coordinates to polar ones*:

$$\rho = \sqrt{x^2 + y^2},$$
$$\varphi = arctg\frac{y}{x}.$$

Note: The last formula gives two values of the angle φ, since it varies from 0 to 2π. From these two values of the angle one has to take one that satisfies Cartesian coordinates x and y.

1.3. Straight line on a plane

The general equation of a straight line is

$$A \cdot x + B \cdot y + C = 0. \tag{1.1}$$

Let's examine the equation (1.1):

– for $C = 0$ equation (1.1) has the form $A \cdot x + B \cdot y = 0$, that is, the straight line passes through the origin of coordinates;

– for $B = 0$ equation (1.1) has the form $A \cdot x + C = 0$, that is, a straight line parallel to the axis Ox;

– for $A = 0$ equation (1.1) has the form $B \cdot y + C = 0$, that is, a straight line parallel to the axis Oy;

– when $B = 0$, $C = 0$ equation (1.1) has the form $A \cdot x = 0$, that is, we obtain the equation of the axis Oy;

– when $A = 0$, $C = 0$ equation (1.1) has the form $B \cdot y = 0$, that is, we obtain the equation of the axis Ox.

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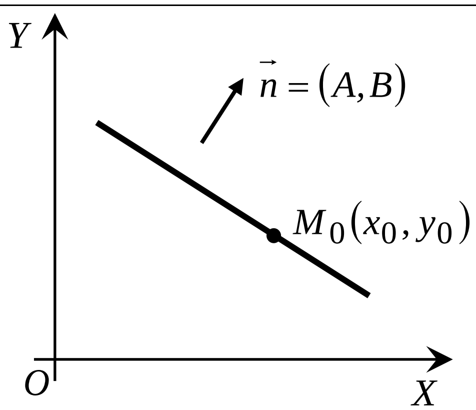

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<td>
1. Equation of a straight line passing through a given point $M_0(x_0, y_0)$ and perpendicular to the vector $\vec{n} = (A, B)$ (normal vector of a straight line):

$$A \cdot (x - x_0) + B \cdot (y - y_0) = 0.$$

2. The general equation of a straight line:

$$A \cdot x + B \cdot y + C = 0, \quad (A^2 + B^2 \neq 0)$$
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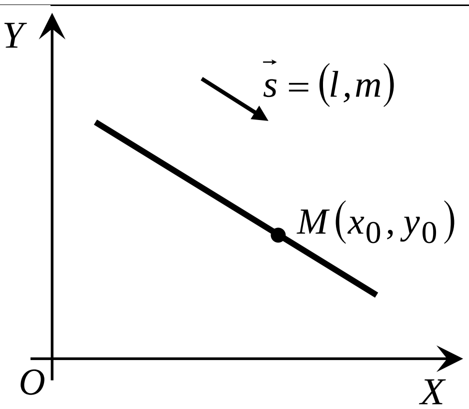

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3. The canonical equation of a straight line (the equation of a straight line passing through a given point $M_0(x_0, y_0)$ and parallel to the vector $\vec{s} = (l, m)$ (directional vector of a straight line)):

$$\frac{x - x_0}{l} = \frac{y - y_0}{m}.$$

4. Parametric equation of a straight line:

$$\begin{cases} x = x_0 + t \cdot l \\ y = y_0 + t \cdot m \end{cases}$$
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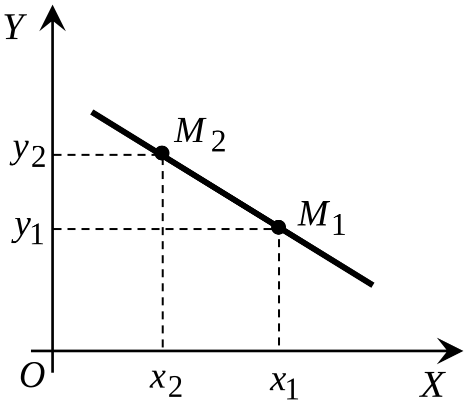
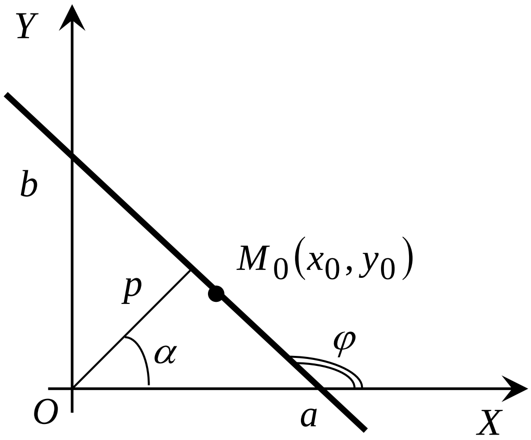


5. The e*quation of a straight line that passes through two given points* $M_1(x_1, y_1)$ *and* $M_2(x_2, y_2)$*:*

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1},$$

$$\left(k = \frac{y_2 - y_1}{x_2 - x_1}\right)$$



6. The equation of a straight line in the segments on the axes:

$$\frac{x}{a} + \frac{y}{b} = 1$$

a – a segment that cuts straight line on the abscissa axis, *b* – on the ordinate axis.
The general equation of line gives

$a = -\frac{C}{A}; b = -\frac{C}{B}$

7. The equation of a straight line with an angular coefficient:

$$y = k \cdot x + b, \ (k = tg\varphi).$$

8. The equation of a straight line passing in a given direction through a given point $M_0(x_0, y_0)$ *(link equation)*:

$$y - y_0 = k \cdot (x - x_0).$$

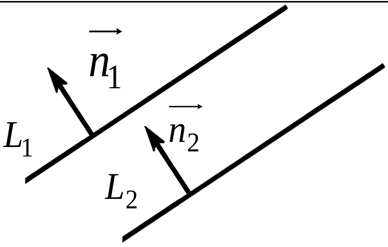
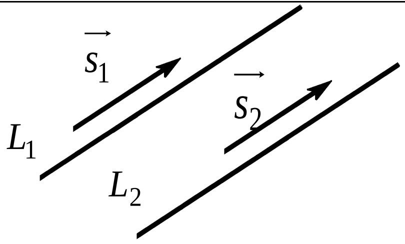
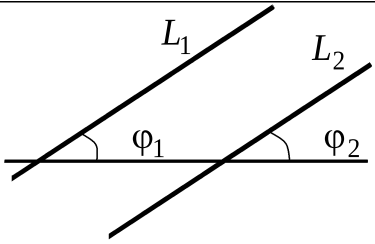
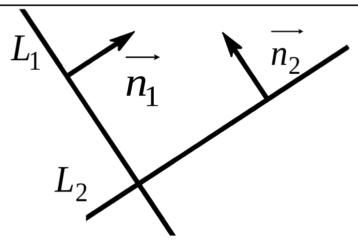
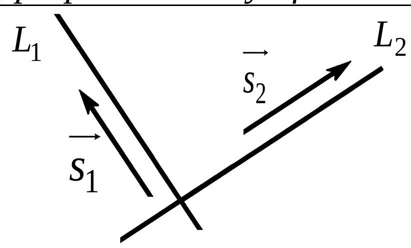
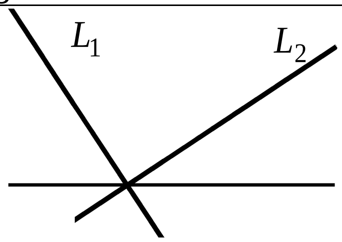
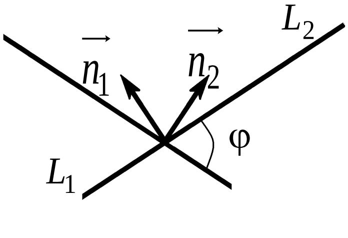
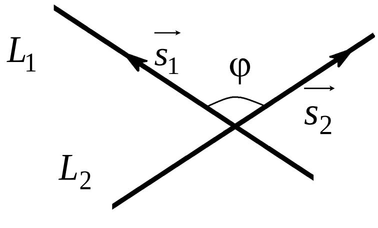
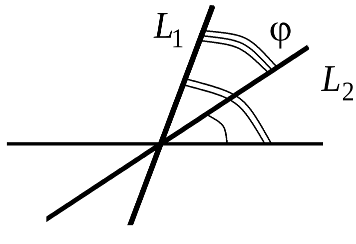
9. The normal equation of a straight line:

$$x \cdot \cos\alpha + y \cdot \sin\alpha - p = 0$$

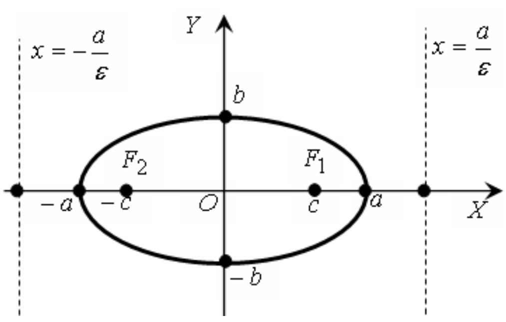
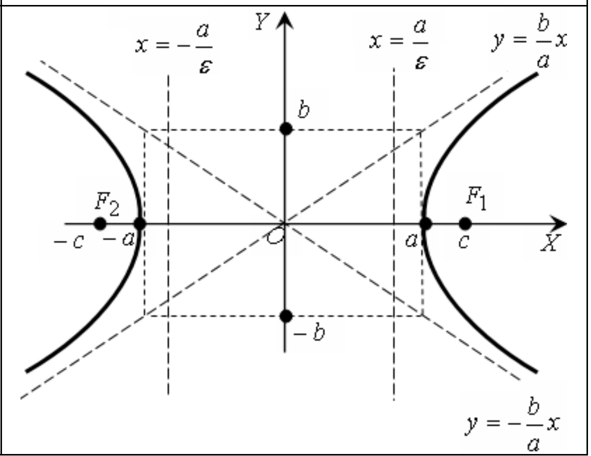
Distance from point $M_0(x_0, y_0)$ to line:

A straight line given by the normal equation
$d_M = \lvert x_0 \cdot \cos\alpha + y_0 \cdot \sin\alpha - p \rvert$
A straight line given by the general equation
$d_M = \dfrac{\lvert A \cdot x_0 + B \cdot y_0 + C \rvert}{\sqrt{A^2 + B^2}}$

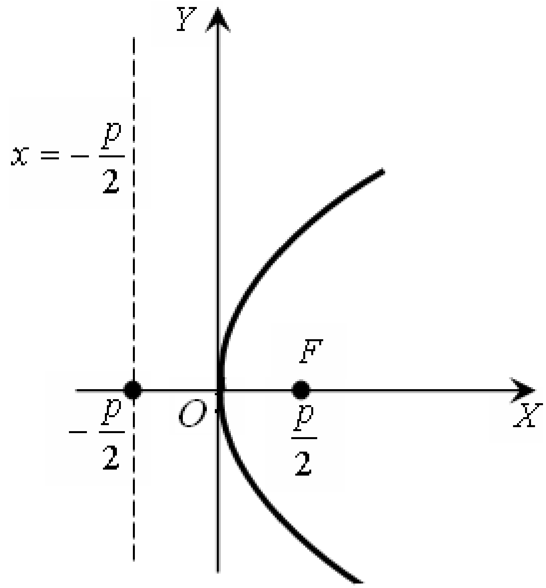
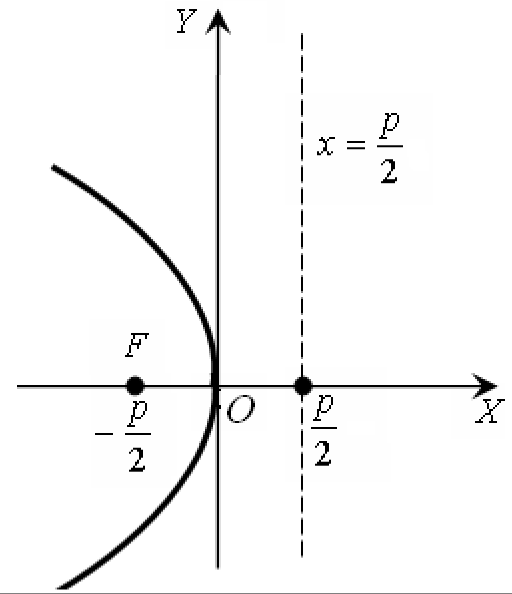
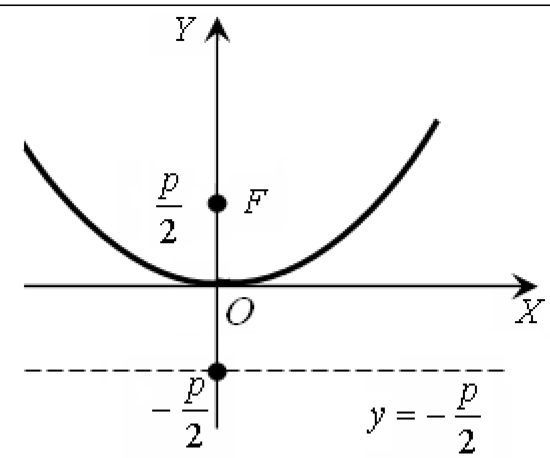
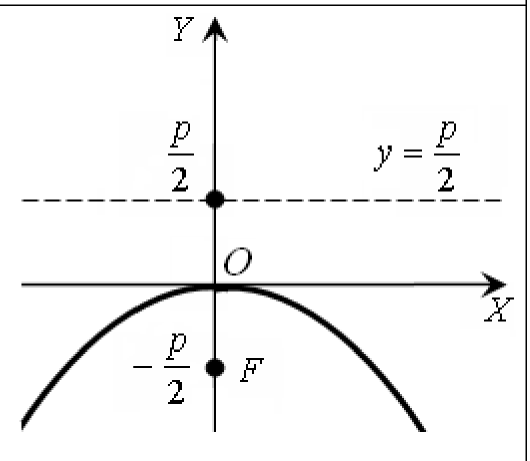
Mutual arrangement of two straight lines on a plane

General equation	Canonical equation	Equation with an angular coefficient
$L_1 : A_1x + B_1y + C_1 = 0 :$ $\overrightarrow{n_1} = (A_1; B_1)$ $L_2 : A_2x + B_2y + C_2 = 0 :$ $\overrightarrow{n_2} = (A_2; B_2)$	$L_1 : \dfrac{x - x_1}{l_1} = \dfrac{y - y_1}{m_1} :$ $\overrightarrow{s_1} = (l_1; m_1)$ $L_2 : \dfrac{x - x_2}{l_2} = \dfrac{y - y_2}{m_2} :$ $\overrightarrow{s_2} = (l_2; m_2)$	$L_1 : y = k_1x + b :$ $k_1 = tg\varphi_1$ $L_2 : y = k_2x + b :$ $k_2 = tg\varphi_2$
1. The condition of the parallelism of two straight lines:		
 $\dfrac{A_1}{A_2} = \dfrac{B_1}{B_2}$	 $\dfrac{l_1}{l_2} = \dfrac{m_1}{m_2}$	 $k_1 = k_2$
2. The condition of perpendicularity of two straight lines:		
 $A_1 \cdot A_2 + B_1 \cdot B_2 = 0$	 $l_1 \cdot l_2 + m_1 \cdot m_2 = 0$	 $k_1 \cdot k_2 = -1$
3. The angle between two straight lines:		
 $\cos\varphi = \dfrac{A_1 \cdot A_2 + B_1 \cdot B_2}{\sqrt{A_1^2 + B_1^2} \cdot \sqrt{A_2^2 + B_2^2}}$	 $\cos\varphi = \dfrac{l_1 \cdot l_2 + m_1 \cdot m_2}{\sqrt{l_1^2 + m_1^2} \cdot \sqrt{l_2^2 + m_2^2}}$	 $tg\varphi = \left\lvert \dfrac{k_2 - k_1}{1 + k_1 \cdot k_2} \right\rvert$

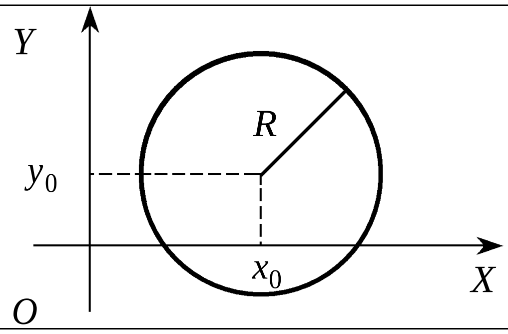
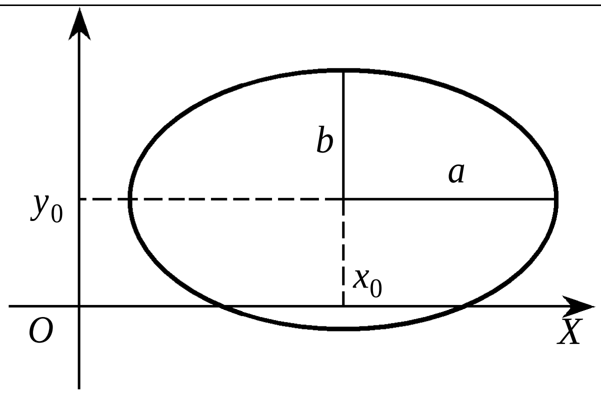
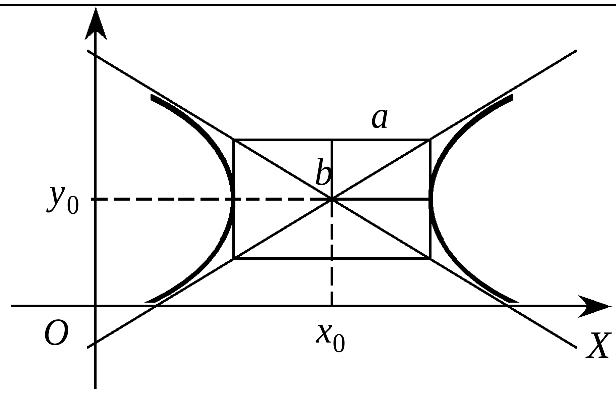
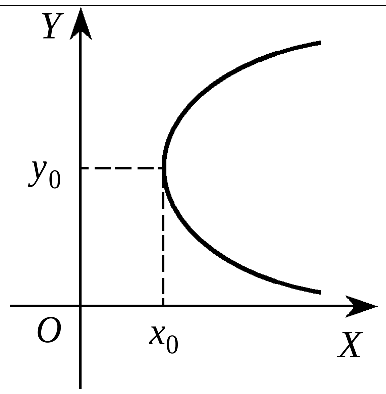
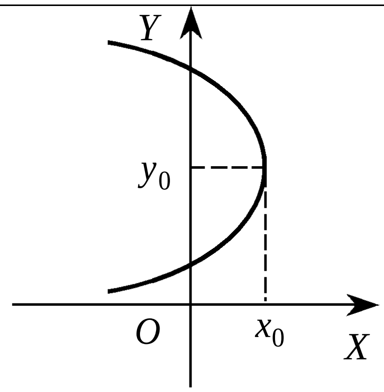
1.4. Second-order curves

Curve	**Ellipse** is the geometric location of the points of the plane, the sum of the distances of which from two given points of this plane, called *focuses*, is constant and greater then distance between the focuses	**Hyperbola** is the geometric location of the points of the plane, for which the module of difference of distances from two given points of this plane, called *focuses*, is constant and smaller distance between the focuses
	Ellipse with focuses on axis Ox	*Hyperbola with focuses on axis* Ox
Equation	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ a>b$	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$
Semiaxes, ($2a$, $2b$ – axes)	a – big semiaxis b – small semiaxis	a – real; b – imaginary semiaxis
Focal length	$c=\sqrt{a^2-b^2}$	$c=\sqrt{a^2+b^2}$
Focuses	$F_1(c;0);\ F_2(-c;0)$	$F_1(c;0);\ F_2(-c;0)$
Eccentricity	$\varepsilon=\frac{c}{a},\quad 0<\varepsilon<1$	$\varepsilon=\frac{c}{a},\quad \varepsilon>1$
Directrix formula	$x=\pm\frac{a}{\varepsilon}\left(x=\pm\frac{a^2}{c}\right)$	$x=\pm\frac{a}{\varepsilon}\left(x=\pm\frac{a^2}{c}\right)$
Asymptotes	–	$y=\pm\frac{b}{a}x$
Focal radii	$r_1=F_1M=a-\varepsilon\cdot x_M$ – right; $r_2=F_2M=a+\varepsilon\cdot x_M$ – left; $r_1+r_2=2a$	$r_1=F_1M=\lvert a-\varepsilon\cdot x_M\rvert$ – right; $r_2=F_2M=\lvert a+\varepsilon\cdot x_M\rvert$ – left; $r_1-r_2=2a$
Picture		

Parabola is called the geometric space of points of a plane equidistant from a given point, which is called a *focus*, and from a given line, called the *directrix* and does not pass through the focus.

Parabolas symmetric with respect to the axis Ox		
Equation	$y^2 = 2 \cdot p \cdot x$	$y^2 = -2 \cdot p \cdot x$
Focuses	$F\left(\frac{p}{2};0\right)$	$F\left(-\frac{p}{2};0\right)$
Directrix formula	$x = -\frac{p}{2}$	$x = \frac{p}{2}$
Picture		
Parabolas symmetric with respect to the axis Oy		
Equation	$x^2 = 2 \cdot p \cdot y$	$x^2 = -2 \cdot p \cdot y$
Focuses	$F\left(0;\frac{p}{2}\right)$	$F\left(0;-\frac{p}{2}\right)$
Directrix formula	$y = -\frac{p}{2}$	$y = \frac{p}{2}$
Picture		

Displaced curves $O'(x_0; y_0)$

<table>
<tr><td>Circle $(x-x_0)^2+(y-y_0)^2=R^2$</td><td>
</td></tr>
<tr><td>Ellipse $\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=1$</td><td>Hyperbola $\frac{(x-x_0)^2}{a^2}-\frac{(y-y_0)^2}{b^2}=1$</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td colspan="2">Parabolas</td></tr>
<tr><td>$(y-y_0)^2=2\cdot p\cdot(x-x_0)$</td><td>$(y-y_0)^2=-2\cdot p\cdot(x-x_0)$</td></tr>
<tr><td>
</td><td>
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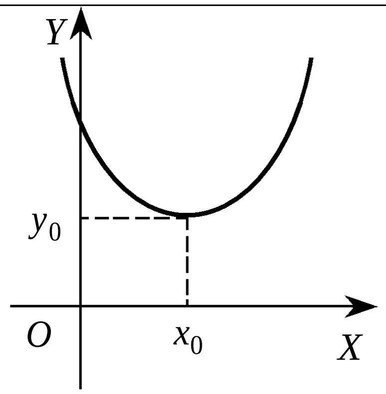
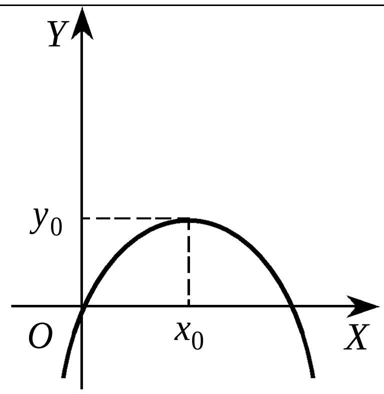
<tr><td>
</td><td>
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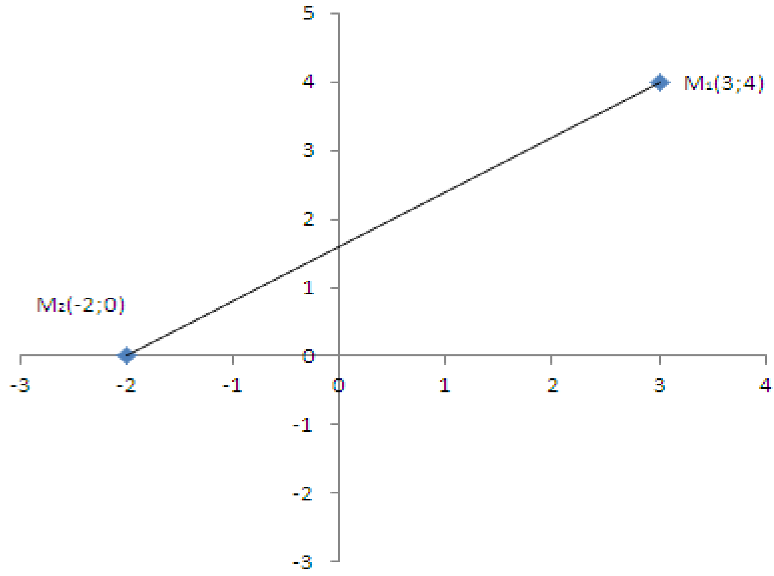
</table>

Example 1. Given points $M_1(3;4)$, $M_2(-2;0)$. Find the distance between them.

Solution. By the formula $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$d=\sqrt{(-2-3)^2+(0-4)^2}=\sqrt{25+16}=\sqrt{41}$.



Example 2. The segment *AB*, wich connect points *A* (2, 5) and *B* (4, 9), divide in the ratio 1:3.

Solution. By the condition of the problem, it is necessary to find the coordinates of the point *C*, which divides the segment *AB* in relation $\lambda=\frac{1}{3}$.

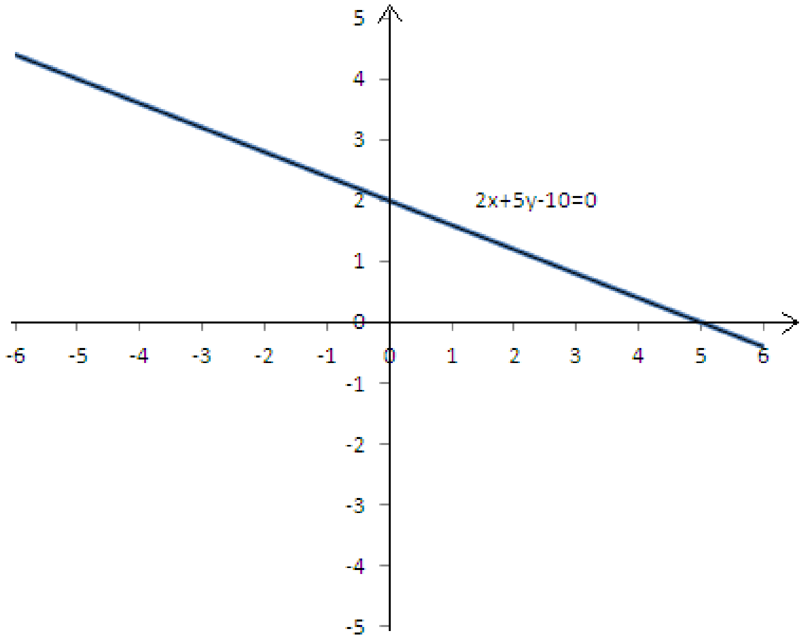
The point *A*(2;5) we consider as the beginning of the segment, and the point *B*(4;9) – as the end. In formulas $x=\frac{x_1+\lambda x_2}{1+\lambda}$; $y=\frac{y_1+\lambda y_2}{1+\lambda}$ x and y are unknown coordinates of the point *C*, x_1 and y_1 – are coordinates of the point *A*, x_2 and y_2 – are coordinates of the point *B*, $\lambda=\frac{1}{3}$. Given x_1=2, x_2=4, y_1=5, y_2=9.

So $x=\frac{2+\frac{1}{3}\cdot 4}{1+\frac{1}{3}}=\frac{5}{2}$, $y=\frac{5+\frac{1}{3}\cdot 9}{1+\frac{1}{3}}=6$.

The point *C* has coordinates $C\left(\frac{5}{2};6\right)$.

Example 3. Write a straight line equation $2x+5y-10=0$ in segments and build this straight line.

Solution. Let's rewrite the equation in form $2x+5y=10$, since $\frac{x}{5}+\frac{y}{2}=1$, so $a=5$, $b=2$. We set off the coordinates of the segments $a=5$, $b=2$ on the axes and through their ends (5;0) i (0;2) draw the straight line.



Example 4. Rectangular coordinates of the point A are given: x=1, y=1. Find its polar coordinates.

Solution. By the formulas of the transition from Cartesian coordinates to polar ones:

$$\rho=\sqrt{x^2+y^2},\ \varphi=arctg\frac{y}{x}$$

we find $\rho=\sqrt{1^2+1^2}=\sqrt{2}$, $\varphi=arctg\frac{1}{1}=\frac{\pi}{4}$. Consequently, the polar coordinates of this point $\rho=\sqrt{2}$, $\varphi=\frac{\pi}{4}$.

Example 5. Specify the feature in the location of the straight lines relative to the coordinate axes

1) 2x-5y=0; 2) 3x-2=0; 3) 7y+12=0; 4) 5x=0; 5) 3y=0.

Solution. 1) The straight line 2x-5y=0 passes through the origin of coordinates, since its equation does not contain a free member.

2) The straight line 3x-2=0 is parallel to the axis Oy (its equation does not contain coordinates y).

3) The straight line 7y+12=0 is parallel to the axis Ox (its equation does not contain coordinates x).

4) The straight line 5x=0 coincides with the axis Oy (its equation can be rewritten in the form x=0).

5) The straight line 3y=0 coincides with the axis Ox (its equation can be rewritten in the form y=0).

Example 6. Write the general equation of the straight line $4x-3y+12=0$ as: 1) with an angular coefficient; 2) in the segments on the axes.

Solution. 1) The straight line equation with angular coefficient has the form $y=k\cdot x+b$. To get a given equation in this form, let's solve it relative y: 3y=4x+12,

$y=\frac{4}{3}x+4$. The angular coefficient of the straight line is $k=\frac{4}{3}$, and the length of the segment that the straight line cuts on the ordinate axis is b=4.

2) The straight line equation in the segments on the axes has the form $\frac{x}{a}+\frac{y}{b}=1$. To determine the size of the segments that the given line $4x-3y+12=0$ cuts, we use the formulas $a=-\frac{C}{A}; b=-\frac{C}{B}$.

From equation we get A=4, B=-3, C=12. $a=-\frac{12}{4}=-3$ $b=-\frac{12}{-3}=4$.

Thus, our equation in the segments on the axes have the form

$$\frac{x}{-3}+\frac{y}{4}=1$$

Example 7. Make an equation of a straight line passing through a point (5;0) perpendicular to the straight line $-3x+2y-6=0$.

Solution. We write the equation in terms of the angular coefficient:

$$y=\frac{3}{2}x+2.$$

$\kappa_1=\frac{3}{2}$. Since straight lines are perpendicular, $k_1\cdot k_2=-1$, thus $k_2=-\frac{1}{\kappa_1}=-\frac{2}{3}$.

Consequently, the equation of the desired line according to the formula $y-y_0=k\cdot(x-x_0)$ has the form $y=-\frac{2}{3}(x-5)$, or $2x+3y-15=0$.

Example 8. Find the point of intersection of straight lines $2x+3y-8=0$ i $x-2y+3=0$.

Solution. By solving the system of equations

$$\begin{cases}2x+3y-8=0\\ x-2y+3=0\end{cases}$$

we get x=1, y=2. Consequently, the straight lines intersect at the point M (1;2).

Example 9. Find the distance from the point M (-1; 2) to the straight line

2x+y-1=0.

Solution. We use the formula of distance from point to line

$$d_M=\frac{|A\cdot x_0+B\cdot y_0+C|}{\sqrt{A^2+B^2}},$$

we get:

$$d_M=\frac{|2\cdot(-1)+1\cdot 2-1|}{\sqrt{2^2+1^2}}=\frac{1}{\sqrt{5}}=\frac{\sqrt{5}}{5}.$$

Example 10. Record the equation of a circle with a center at C(2; -3) and a radius equal to 6.

Solution. Given x_0=2, y_0=-3, R=6. According to equation

$$(x-x_0)^2+(y-y_0)^2=R^2,$$
we get $(x-2)^2+(y+3)^2=36$ or $x^2+y^2-4x+6y-23=0$.

Example 11. Find the coordinates of the center and the radius of the circle

$$x^2+y^2-2x+4y-11=0.$$

Solution. We group members that contain only *x* and only *y*:

$$x^2-2x=(x-1)^2-1$$
$$y^2+4y=(y+2)^2-4$$

The given equation we rewrite in the form

$$(x-1)^2-1+(y+2)^2-4-11=0$$
$$(x-1)^2+(y+2)^2-16=0$$
$$(x-1)^2+(y+2)^2=16$$

Consequently, the center of the circle is at the point (1; -2), and the radius is equal to 4.

Example 12. Make the ellipse equation knowing that:

1) semiaxes are a=6, b=3;

2) the distance between the focuses 2c = 10, and the large axis is 2a=16;

3) the large semiaxis is a=12, and eccentricity is $\varepsilon=0{,}5$

Solution. 1) The canonical ellipse equation has the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Substituting into it a=6, b=3, we get

$$\frac{x^2}{36}+\frac{y^2}{9}=1.$$

2) We have 2c=10; c=5

2a=16; a=8.

To write the equation of an ellipse, one must find a smaller semiaxis b. There is an addiction between the values of a, b, and c in the ellipse $c=\sqrt{a^2-b^2}$, or $b^2=a^2-c^2$. In our case $b^2=64-25=39$, so the equation of the ellipse looks like

$$\frac{x^2}{64}+\frac{y^2}{39}=1.$$

3) a=12; we know, that $\varepsilon=\frac{c}{a}$, in this equation we know value c. To determine it, we obtain the equation $0{,}5=\frac{c}{12}$, thus c=6.

Knowing that a = 12, c = 6, we will use the relation $c=\sqrt{a^2-b^2}$, so we get $b^2=a^2-c^2=144-36=108$.

The equation of the ellipse has the form

$$\frac{x^2}{144}+\frac{y^2}{108}=1.$$

Example 13. Make a canonical equation of hyperbola, if the distance between its vertices is 20, and the distance between the focuses is 30.

Solution. The vertices of hyperbola lie on its real axis. It is given that 2a=20, 2c=30. So, a=10, c=15.

Values a, b and c linked by the relation in hyperbola $c=\sqrt{a^2+b^2}$, hence $b^2=c^2-a^2=225-100=125$. Substituting the values a and b in the canonical equation of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, we have

$$\frac{x^2}{100}-\frac{y^2}{125}=1$$

Example 14. Find eccentricity of the hyperbola $9x^2-16y^2=144$.

Solution. We present an equation of hyperbola in canonical form

$$\frac{x^2}{16}-\frac{y^2}{9}=1$$

hence a=4, b=3, we find c

$c=\sqrt{a^2+b^2}=\sqrt{16+9}=\sqrt{25}=5$.

The eccentricity is $\varepsilon=\frac{c}{a}$, $\varepsilon=\frac{5}{4}$.

Example 15. Parabola $y^2=2px$ passes through the point A(2; 4). Determine its parameter p

Solution. We substitute in the parabola equation instead of the coordinates of the point coordinates A(2;4). We get

$4^2=2p\cdot 2$, $16=4p$, p=4.

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